SINGLE-PHASE OPERATION OF THREE-PHASE MOTORS*

By P. VENKATA RAO AND CHANDRA SEKHAR GHOSH

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ABSTRACT. Single-phase operation of three-phase motors has been a subject of interest to both design and operating engineers. A large amount of literature exists, in most of which the method of symmetrical components analysis has been used, for the analysis of performance of three-phrse motors with unbalanced voltages. In this paper the method of Dyadic circuit analysis developed by Sah is applied to the problem of operation of three-phase motors from single-phase supply and the necessary relationships have been devoloped of the complete performance characteristics from standstill to synchronous speed with auxiliary impedances in circuit.

A considerable amount of work has been done on the subject of single-phase induction motors which use split-phase starting. The existing literature on these machines deals with the auxiliary impedances required and the torque developed. The use of auxiliary impedances in the single-phase operation of polyphase motors is also well known.

When a polyphase motor is operated from a single-phase supply, and auxiliary impedances are used to shift the phase currents and the motor terminal voltages so that the currents and voltages of the motor are properly balanced both as to phase and magnitude, the motor will operate as though it were supplied from its normal polyphase line. The two common methods in use are the series impedance method and the shunt impedance method. By a judicious choice of the resistance and reactance values of the auxiliary impedances it is possible to have reasonably good balancing, both as regards phase and magnitude of the currents and voltages. It happens quite often that the external impedances are used only for starting purposes, the motor being kept running with two of its terminals connected direct to the single-phase line, the third terminal being left open.

Due to the fact that the motor current and power factor both change as the speed changes, the auxiliary impedances required also must change with the motor speed. Of interest, therefore, is the determination of the performance characteristics of a three-phase motor on single-phase supply over the complete speed range from standstill to synchronous speed with the auxiliary impedance in circuit.

Lumin (1936) in his paper, outlines a method that is quite general but the utility of his method is limited by the tedious algebra involved. Reed and Koopman (1936) have presented an analysis which is neither so tedious nor highly involved. They use the exact equivalent circuit and the analytical

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determination of the characteristics which are familiar to all designers of induction motors. In their paper they have also applied the analysis to the performance of some motors under various conditions of starting and operating with auxiliary capacitance.

Tracy and Wyss (1935) have analysed the problem of split-phase starting of three-phase motors and have arrived at some very useful conclusions. They determined the best values of resistance and reactance for series and shunt methods of split-phase starting. In the first method, the starting current is not restricted to any particular value as in the second to obtain the maximum starting torque. In the analysis, a direct solution is visualised whereby the optimum values of resistance, reactance or capacitance can be arrived at for maximum starting torque with an arbitrarily fixed starting current in the line. Thacker and Gopalakrishna (1948), in their paper have extended the work of Tracy and Wyss using the same method of symmetrical components. In their paper, methods of calculations are presented for the determination of the complete performance characteristics from standstill to synchronous speed of a three-phase motor on a single-phase supply with the auxiliary impedances in circuit. They have set up equations taking into account generalized impedances so that the equations may be applicable to any combination of resistances, reactances and capacitances. However, the final relations for the line currents, voltages and torque are expressed in terms of the sequence impedances. The vector diagrams drawn for the equivalent circuit of the motor with the auxiliary impedances are also incomplete. Further, the steps required in arriving at the final relationships are also long and involved.

The Dyadic circuit analysis method developed by Sah (1939) offers an extremely simple and short method of calculating the performance of three phase motors under unbalanced voltages. In this paper, simple calculations are given by the method of Dyadic algebra for the exact detemination of the complete performance characteristics from standstill to synchronous speed of a three-phase motor run from single-phase supply with auxiliary impedances in circuit. The correct vector diagrams for the currents and voltages for conditions with series impedance and shunt impedance in circuit are given. Expressions for the starting torque and starting current have been obtained in terms of phase impedances. This is an advantage compared to the other formula developed previously in which the relationships have been obtained in terms of the sequence impedances. By simple substitution of the sequence impedance values in the final formula for the current, the relationships developed here are shown to agree with those developed by other authors.

Finally, as a matter of interest, the method has been applied to the most commonly employed system in which a capacitance is placed in shunt across two of the phases, while no other impedance is used in the other branch. Expressions for the line current, sequence currents and torque are developed for this arrangement.

SERIES IMPEDANCE METHOD OF OPERATION

Figure 1(*a*) gives the equivalent circuit with series impedances and the vector diagram for the currents and voltages with this arrangement is given in figure 1(*b*).

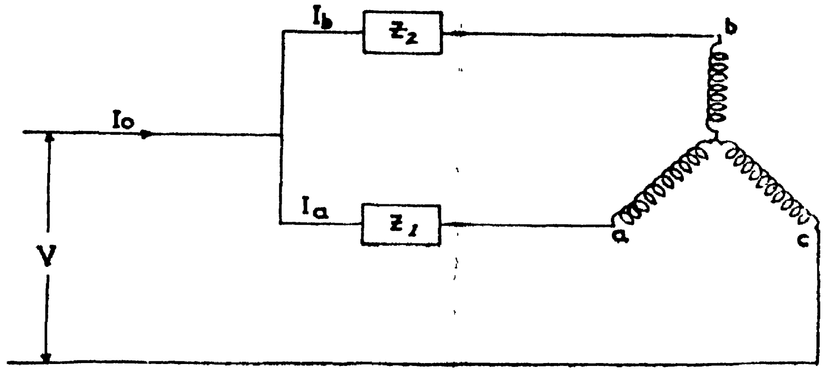

FIG. 1(*a*)

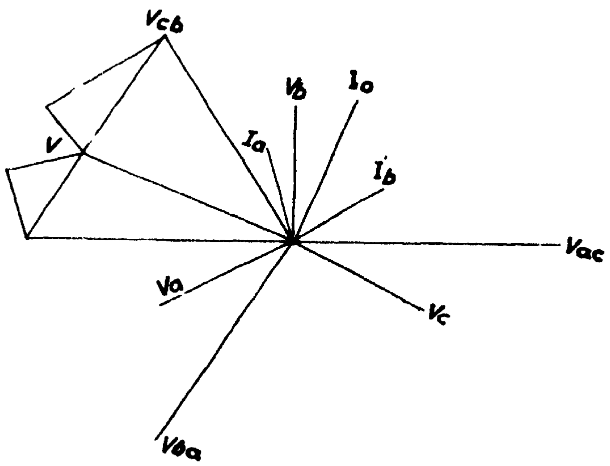

FIG. 1(*b*)

Vector diagram of series impedance circuit

The fundamental voltage equations are

$$\left.\begin{aligned} V_a &= AI_a + CI_b + BI_c \\ V_b &= BI_a + AI_b + CI_c \\ V_c &= CI_a + BI_b + AI_c \end{aligned}\right\} \qquad \dots \quad (1)$$

where V_a, V_b, and V_c are the voltages to the neutral in the respective phases.

Referring to figure 1 (*a*) and by use of Kirchoff's laws,

$$\left.\begin{aligned} I_o &= I_a + I_b \\ I_c &= -I_o \end{aligned}\right\} \qquad \dots \quad (2)$$

$$V = I_a Z_1 - V_{ac} = I_b Z_2 - V_{bc} \qquad \dots \quad (3)$$

Since power input is VA,

$$\text{Efficiency} = \frac{T(1-s)-(F+W)}{VA}$$

SHUNT IMPEDANCE METHOD OF OPERATION

The schematic circuit arrangement is shown in figure 2(a). Figure 2(b) gives the vector diagram for the currents and voltages in the circuit of figure 2(a).

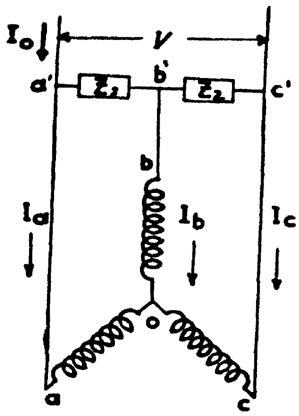

FIG 2(a)

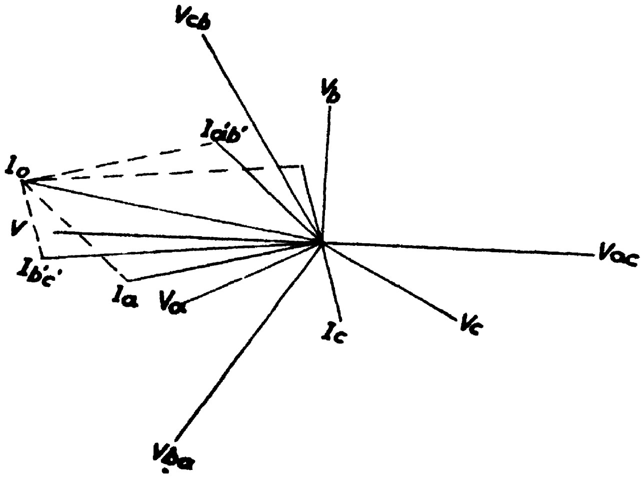

FIG 2(b)
Vector diagram of shunt impedance circuit

By use of Kirchoff's law,

$$I_b = -(I_a + I_c)$$

$$\left. I_o = I_a + \frac{V_{ba}}{Z_1} = \frac{V_{cb}}{Z_2} - I_c \right\} \quad \ldots \quad (17)$$

and
$$V = V_{ca} \quad \ldots \quad (18)$$

Hence, by using the fundamental voltage equations (1)

$$\left.\begin{aligned} V_a &= I_a(A-C) + I_c(B-C) \\ V_b &= I_a(B-A) + I_c(C-A) \\ V_c &= I_a(C-B) + I_c(A-B) \end{aligned}\right\} \quad \ldots \quad (19)$$

Using the same notations K, M and N as before, from equations (18) and (19), it follows

$$V = V_{ca} = I_a(-N) + I_c M$$

also

$$\left.\begin{aligned} V_{ba} &= I_a K + I_c(-N) \\ V_{cb} &= I_a M + I_c(-K) \end{aligned}\right\} \quad \ldots \quad (18a)$$

From equation (17) therefore

$$Z_1 I_o = Z_1 I_a + V_{ba} = Z_1 I_a + I_a K + I_c(-N) = I_a(K+Z_1) - I_c N \quad \ldots \quad (20)$$

Similarly,
$$Z_2 I_o = I_a M - I_c(K+Z_2) \quad \ldots \quad (21)$$

Solving for I_a and I_c

$$\left.\begin{aligned} I_a &= \frac{I_o\{Z_1(K+Z_2)-NZ_2\}}{(K+Z_1)(K+Z_2)-MN} \\ I_c &= \frac{I_o\{MZ_1-Z_2(K+Z_1)\}}{\{(K+Z_1)(K+Z_2)-MN)\}} \end{aligned}\right\} \quad \ldots \quad (22)$$

Substituting these values of I_a and I_c in (18*a*)

$$V=I_o\left[\frac{-N\{Z_1(K+Z_2)-NZ_2\}}{\{(K+Z_1)(K+Z_2)-MN\}}+\frac{M\{MZ_1-Z_2(K+Z_1)\}}{\{(K+Z_1)(K+Z_2)-MN\}}\right.$$

Hence

$$I_o=\frac{V\{(K+Z_1)(K+Z_2)-MN\}}{[\{N^2Z_2-NZ_1(K+Z_2)\}+\{M^2Z_1-MZ_2(K+Z_1)\}]} \quad \ldots \quad (23)$$

It is thus possible to predetermine the value of the line current from the known values of the auxiliary impedances and the phase impedance. Since K, M. N can be easily determined for any value of slip, it is possible to determine the current I_o at any slip.

The conditions at standstill are,

$$Z_+ + Z_- = Z \; ; \; B=C$$

$$M=N=B-A=-Z$$

$$K=2Z$$

Hence, by substitution in (23)

$$I_{st}=\frac{V\{(K+Z_1)(K+Z_2)-M^2\}}{\{N^2Z_2-NZ_1(K+Z_2)+M^2Z_1^2-MZ_2(K+Z_1)\}} \quad \ldots \quad (24)$$

In terms of symmetrical components

$$MN=(a^2Z_+ + aZ_-)(aZ_+ + a^2Z_-)=Z_+^2+Z_-^2-Z_+Z_-$$

$$M^2=aZ_+^2+a^2Z^2_- + 2Z_+Z$$

$$N^2=a^2Z_+^2+aZ_-^2+2Z_+Z$$

$$K^2=Z_+^2+Z_-^2+2Z_+Z_-$$

$\therefore$ From (23)

$$I_o=\frac{V[Z_1Z_2+(Z_1+Z_2)(Z_+ + Z_-)+3Z_+Z_-]}{[(Z_+ + Z_-)Z_1Z_2+3Z_+Z_-(Z_1+Z_2)]} \quad \ldots \quad (23a)$$

and from (24)

$$I_{st}=\frac{V[Z_1Z_2+2Z(2Z_1+Z_2)+3Z^2]}{[2ZZ_1Z_2+3Z^2(Z_1+Z_3)]} \quad \ldots \quad (24a)$$

also

$$I_a=I_o\frac{Z_1(Z_2+2Z)+ZZ_2}{(Z_1+2Z)(2Z+Z_2)-Z^2}$$

$$I_c=I_o\frac{-2Z_1-Z_2(Z_1+2Z)}{(Z_1+2Z)(Z_2+2Z)-Z^2}$$

and the positive and negative sequence currents are

$$\left.\begin{aligned} I_+ &= \frac{I_a(1-a)+I_c(a^2-a)}{\sqrt{3}} \\ I_- &= \frac{I_a(1-a^2)+I_c(a-a^2)}{\sqrt{3}} \end{aligned}\right\} \quad \ldots \quad (25)$$

as before, torque $$T = 3\left[\frac{r'}{s}\left|I_+\right|^2 - \frac{r''}{(2-s)}\left|I_-\right|^2\right] \quad \ldots \quad (26)$$

A particular case

The special case in which a capacitance is connected across two of the phases while there is no impedance connected across the other side is shown diagrammatically in figure 3(*a*) and the corresponding vector diagram for currents and voltages is given in figure 3(*b*).

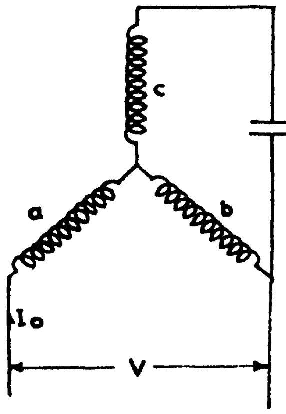

FIG. 3(*a*)

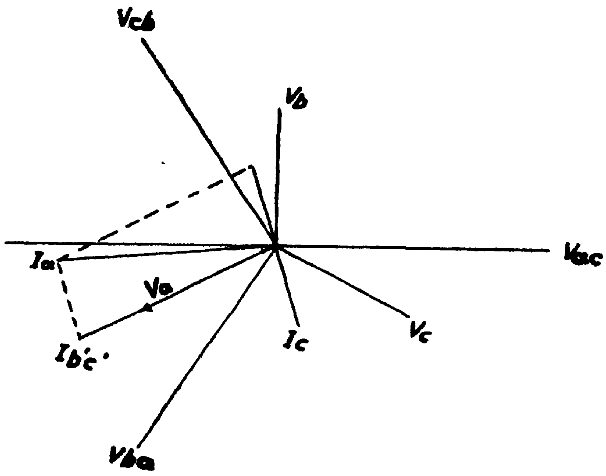

FIG. 3(*b*)
Vector diagram of capacitor in shunt circuit

As shown in figure 3(*a*), the conditions in this case are

$$Z_1 = \infty \text{ and } Z_2 = -jX$$

Hence from equation (24*a*), after dividing the numerator and denominator by Z_1

$$I_{st} = V\left[\frac{Z_2 + 2Z + (Z/Z_1)(3Z + 2Z_2)}{2ZZ_2 + 3Z^2 + 3Z^2Z_2/Z_1}\right]$$

Now, substituting the values of Z_1 and Z_2 in this case,

$$I_{st} = \frac{V(2Z - jX)}{Z(3Z - 2jX)} \quad \ldots \quad (27)$$

In the same way, substituting the values for Z_1 and Z_2 in eqn. (23*a*) after proper transformation, the current during running condition is

$$I_o = \frac{V(Z_+ + Z_- - jX)}{[3Z_+Z_- - jX(Z_+ + Z_-)]} \quad \ldots \quad (28)$$

The values of I_+ and I_- are the same as given in equation (25). The currents in the other branches at standstill condition are also determined as below.

Since $I_a = I_o$, and $Z_2 = -jX$, in this case, and remembering that $M = -Z$ and $K = 2Z$ at standstill, from equation (21)

$$I_c = I_o\frac{(M - Z_2)}{(K + Z_2)} = I_o\left|\frac{-(Z + Z_2)}{(2Z + Z_2)}\right| = I_o\left|\frac{-(Z - jX)}{(2Z - jX)}\right|.$$

After substituting for I_o, the expression obtained in (27)

$$I_c = V\left\{\frac{-Z + jX}{Z(3Z - 2jX)}\right\} \quad \ldots \quad (29)$$

Substituting for I_a and I_c in equations (25)

$$\sqrt{3}I_+ = V\left\{\frac{(2Z - jX)(1 - a)}{Z(3Z - jX)} + \frac{(Z + jX)(a^2 - a)}{Z(3Z - 2jX)}\right\}$$

$$= V\left\{\frac{3Z + jX(a^2 - 1)}{Z(3Z - 2jX)}\right\} \quad \ldots \quad (30a)$$

Similarly

$$\sqrt{3}I_- = V\left\{\frac{3Z + jX(a - 1)}{Z(3Z - 2jX)}\right\} \quad \ldots \quad (30b)$$

torque

$$T = 3\left[\frac{r'}{s}\left|I_+\right|^2 - \frac{r''}{(2 - s)}\left|I_+\right|^2\right]$$

Symbols used in the paper

V_a, V_b, V_c — Voltages of the three respective phases to neutral

Z_1, Z_2 — External auxiliary impedances

T — Torque developed in synchronous watts

P — Power converted to mechanical form

$F + W$ — Friction and windage loss in watts

Z — Machine phase impedance

Z_+, Z_- — Positive and negative sequence impedances

A, B, C — Respective phase impedance parameters of the machine

I_o — Single-phase line current

I_+, I_- — Sequence components of current

V — Applied single-phase voltage.

The steady state values for the machine constants are :

$$A = r_a + jX_a + \frac{3X_m^2}{4}\left\{\frac{s}{(r_x + jsX_x)} + \frac{2-s}{\{r_x + j(2-s)X_x\}}\right\}$$

$$B = -jX_{ab} + \frac{3X_m^2}{4}\left\{\frac{a^2 s}{(r_x + jsX_x)} + \frac{a(2-s)}{\{r_x + j(2-s)X_x\}}\right\}$$

$$C = -jX_{ab} + \frac{3X_m^2}{4}\left\{\frac{as}{(r_x + jsX_x)} + \frac{a^2(2-s)}{\{r_x + j(2-s)X_x\}}\right\}$$

DEPARTMENT OF POWER ENGINEERING
INDIAN INSTITUTE OF SCIENCE, BANGALORE.

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